Name: STANFORD Date:______________

NV Calculus – Pquiz, Chapter 3.1-3.4: Derivatives

Find the derivatives of the following functions (5 pts.)

1. $f(x) = x^4$

$\boxed{f'(x) = 4x^3}$

2. $y = 6x^3$

$\boxed{\frac{dy}{dx} = 18x^2}$

3. y = 4x3 + 2

$\frac{dy}{dx} = \boxed{12x^2}$

4. $f(x) = 5x^3 - 4x^2 + 2x - 1$

$\boxed{f'(x) = 15x^2 - 8x + 2}$

5. $f(x) = (x^3 + 1)(x^4 - 7)$

$f'(x) = (3x^2)(x^4 - 7) + 4x^3(x^3 + 1)$

$= 3x^6 - 21x^2 + 4x^6 + 4x^3$

$= \boxed{7x^6 + 4x^3 - 21x^2}$

6. $y = \frac{1}{x} = x^{-1}$

$\frac{dy}{dx} = -x^{-2} = \boxed{\frac{-1}{x^2}}$

7. $y = \frac{1}{\sqrt{x}} = x^{-1/2}$

$\frac{dy}{dx} = \frac{-1}{2}x^{-3/2} = \frac{-1}{2x^{3/2}} = \boxed{\frac{-1}{2\sqrt{x^3}}}$

8. $f(t) = \frac{t}{t+2}$

$f'(t) = \frac{(t+2) - t}{(t+2)^2} = \boxed{\frac{2}{(t+2)^2}}$

9. $f(x) = \frac{x^2+1}{3x^3-5}$

$f'(x) = \frac{2x(3x^3-5) - 9x^2(x^2+1)}{(3x^3-5)^2}$

$= \frac{6x^4 - 10x - 9x^4 - 9x^2}{(3x^3-5)^2} = \boxed{\frac{-3x^4 - 9x^2 - 10x}{(3x^3-5)^2}}$

10. $y = \frac{2x+1}{\sqrt{x}} = \frac{2x+1}{x^{1/2}}$

$y' = \frac{2(x^{1/2}) - (2x+1)(\frac{1}{2}x^{-1/2})}{x}$

$y' = \frac{2\sqrt{x} - (\sqrt{x} + \frac{1}{2}x^{-1/2})}{x} = \frac{\sqrt{x} + \frac{1}{2\sqrt{x}}}{x}$

$= \frac{\sqrt{x}}{x} + \frac{1}{2x\sqrt{x}} = \boxed{\frac{\sqrt{x}}{x} + \frac{\sqrt{x}}{2x^2}}$

11. (10pts)Use the definition $m_{tan} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ to determine the slope of the tangent line for

$f(x) = x^2 + 8x$ where $a = 2$. Show all your work!

$m_{tan} = \lim_{x \to 2} \frac{(x^2 + 8x) - (2^2 + 8 \cdot 2)}{x - 2} = \lim_{x \to 2} \frac{x^2 + 8x - 20}{x - 2} = \lim_{x \to 2} \frac{\cancel{(x-2)}(x+10)}{\cancel{(x-2)}}$

$= \lim_{x \to 2}(x + 10) = \boxed{12}$

12. 10pts) Given $f(x) = 3x^2 - 4$ and a = 1, Use the definition $m_{tan} = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$ to find the slope of the tangent line at a. Show all your work!

$\cancel{3} + 6h + 3h^2 \cancel{-3}$

$$\lim_{h\to 0} \frac{3(1+h)^2-4-(3(1^2)-4)}{h} = \lim_{h\to 0} \frac{3(1+2h+h^2)-4+1}{h} = \lim_{h\to 0} \frac{6\cancel{h}+3h^{\cancel{2}}}{\cancel{h}} = \lim_{h\to 0} 6+3h$$

$$\boxed{=6}$$

13. 10pts) Given the function $f(x) = x^3 - 3x^2 - 9x + 6$

a) find the equation of the line tangent to the curve at x = 1

b) find the x coordinates where the slope of the tangent lines are zero

a) $f'(x) = 3x^2 - 6x - 9$

$f'(1) = 3 - 6 - 9 = -12$

slope $= -12$,

$y = f(1) = 1^3 - 3(1^2) - 9(1) + 6 = -5$

$y = -12x + b$

$-5 = -12(1) + 6$

$b = 7$, so

$\boxed{y = -12x + 7}$

b) $f'(x) = 3x^2 - 6x - 9 = 0$

or

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$\boxed{x=3}$ and $\boxed{x=-1}$

14. 10pts)The height of an object is given by the equation $h(t) = 630 + 80t - 16t^2$ where y is position in feet and t is time in seconds

a) find an equation for the velocity as a function of time

$v(t) = \frac{dh}{dt} = \boxed{80 - 32t}$

b) when does the object reach its maximum height?

$v(t) = 0 = 80 - 32t$, $t = \frac{80}{32} = \boxed{2.5s}$

c) what is its maximum height?

$h(\frac{5}{2}) = 630 + 80\frac{5}{2} - 16(\frac{5}{2})^2 = \boxed{730 ft}$

d) what is the acceleration of the object?

$a(t) = \frac{dv}{dt} = \boxed{-32\ ft/s^2}$

15. 10pts)The cost, in dollars, to produce x widgets is given by

$C(x) = 200 + \frac{5}{x} + \frac{x^2}{5}$

a) find an equation for the marginal cost, MC(x)

b) find the marginal cost of producing 13 widgets

c) find the actual cost to produce the 14th widget

a) $MC(x) = \frac{dC}{dx} = \frac{d}{dx}\left[200 + 5x^{-1} + \frac{1}{5}x^2\right]$

$= -5x^{-2} + \frac{2}{5}x = \boxed{\frac{-5}{x^2} + \frac{2}{5}x}$

b) $MC(13) = \frac{-5}{13^2} + \frac{2}{5}(13) = \boxed{5.17...}$

c) $C(14) - C(13) = \cancel{200} + \frac{5}{14} + \frac{14^2}{5} - \left[\cancel{200} + \frac{5}{13} + \frac{13^2}{5}\right] = \boxed{5.37...}$